The Reduced Form of Litigation Selection Models and the Plaintiff’s Win Rate

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Current version: October 11, 2015
First version: September 26, 2013

Abstract

In this paper I develop a new approach—the reduced form approach—to studying the plaintiff’s win rate in one-shot litigation selection models. A reduced form requires three basic elements. First, a joint distribution of plaintiff’s and defendant’s beliefs concerning the probability that the plaintiff would win in the event a dispute were litigated. Second, a conditional win rate function that returns the actual probability the plaintiff would win if the case were litigated, given the parties’ subjective beliefs. Third, a reduced form requires a specification of a litigation rule, which tells us the probability that a case will be litigated given the two parties’ subjective probabilities.

I use the reduced form to prove several general results. First, I show that the plaintiff’s win rate will always equal one-half when certain balance conditions on the conditional win rate function, joint density, and litigation rule all hold. Second, I show that when these balance conditions are systematically violated in the same direction, the plaintiff’s win rate will deviate from one-half in a predictable direction. Third, I prove that any plaintiff’s win rate between 0 and 1 is possible, even in the limit as party information becomes very good, and even without the kind of asymmetric information that Shavell (1996) has suggested is the key to generating this result. Fourth, I show via a simple constructive example that there is no reason to expect the plaintiff’s win rate to move in a predictable direction when legal rules change, contra the optimistic conclusions in Klerman & Lee (2014) concerning the empirical usefulness of plaintiff’s win rate data.

∗For helpful comments and suggestions, I thank Daniel Klerman, Jon Klick, Alex Lee, Rick Brooks, Charles Silver, Steven Salop, Sarath Sanga, David Schleicher, Joshua Teitelbaum, Abe Wickelgren, and participants at ALEA 2014, the Yale-Paris 2 Summer School in Law and Economics, and workshops at Georgetown and the University of Texas.
1 Introduction

In this paper I develop a new approach—the reduced form approach—to studying the plaintiff’s win rate in one-shot litigation selection models. The core of a reduced form is a pair of beliefs—one held by the plaintiff, and one by the defendant—concerning the probability that the plaintiff would win in the event a dispute were litigated. Beyond that, a reduced form involves three basic functions. The first is a function specifying the joint distribution across disputes of the parties’ respective subjective beliefs concerning the probability the plaintiff would win in litigation. The second function is a conditional win rate function, whose value is the actual probability the plaintiff would win if the case were litigated, given the parties’ subjective beliefs. The third function is a litigation rule, which tells us the probability that a case will be litigated given the two parties’ subjective beliefs.

To illustrate a reduced form in action, consider points 1 and 2 in Figure XXX [NOTE TO READERS: I NEED TO ADD THIS FIGURE, BUT FOR NOW JUST LOOK AT POINTS 1 AND 2 IN Figure 2]. These points are, together, an example of what I call a balance pair. I define the concept of a balance pair in section 4, but for now the key point is that results related to the plaintiff’s win rate among litigated cases may be derived from the balance pair-relevant properties of the distribution of party beliefs, the conditional win rate function, and the litigation rule. When these functions all satisfy certain balance conditions, the plaintiff’s win rate among litigated will always be one-half. In the discussion below, I connect this fact to the long-running debate over selection in litigation that started with Priest & Klein (1984). I also offer imbalance conditions that are sufficient for the plaintiff’s win rate to be either greater or less than one-half. These results are important insofar as they help us understand what drives the apparent tendency of the plaintiff’s win rate either toward or away from one-half.1

The reduced form approach allows me to derive other results that upend certain nuggets of received wisdom. In section 5, I use a trivial constructive example to show that the plaintiff’s win rate may equal any value between 0 and 1. While this result echoes Shavell (1996), unlike Shavell I do not rely on asymmetric information; my result holds even when parties disagree and would continue to do so even if both parties knew each other’s true beliefs. Thus, my result shows that asymmetric information is unnecessary to obtain the any-win-rate result. That is important because it weakens the case—argued by Shavell—for relying on asymmetric information to rationalize observed plaintiff’s win rates that are far from one-half. Moreover, in the same section I show that even in a divergent-expectations framework, there is no necessary tendency of the plaintiff’s win rate toward any particular plaintiff’s win rate value—whether one-half or some other value—as the parties’ information becomes more accurate. Results in the literature that do generate such convergences, e.g., Lee & Klerman (2015b), must thus be regarded as special to particular modelling assumptions.

The reduced form approach’s simplicity makes it attractive for use in answering other

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1 On this issue, see also impressive recent derivations in Lee & Klerman (2015b) and Lee & Klerman (2015a).
complicated questions about litigation selection. For example, I use it to show that there is no particular reason to think that the plaintiff’s win rate among litigated cases should move in a predictable direction when legal rules move in the plaintiff’s favor. This result directly contradicts the optimism of Klerman & Lee (2014) concerning the possibility of using changes in the observed win rate to assess the direction of change in legal rules. This is an important issue not just for theoretical inquiry, but also for empirical work, given both suggestions in Klerman & Lee (2014) and the enduring use of empirical win rates to study changes in legal rules.

The next section briefly discusses the literature on litigation selection and the plaintiff’s win rate. Section 3 introduces the basic elements of my reduced form approach. Section 4 introduces two balance conditions that can be used to construct sufficient conditions under which the plaintiff’s win rate among litigated cases will equal exactly one-half. I then show how imbalance conditions can be used to construct sufficient conditions under which the plaintiff’s win rate will not equal one-half. In section 5 I present several results indicating that the plaintiff’s win rate among litigated cases may take on any value, even without asymmetric information, and even when party information converges to being perfect. Further, I show that the selection of cases for litigation creates no general bias toward one-half, even when stakes are symmetric. In section 6, I give a simple illustration of why and how the plaintiff’s win rate might either rise or fall after a change in legal rules, even when the new legal rule is systematically more favorable to one side or the other in litigation. Finally, in section 7, I translate three important litigation selection models into the reduced form terms I introduce here, discussing how and whether those models may be understood in terms of the balance properties defined in section 4. I conclude in section 8.

2 Related Literature

My focus here is on what we can learn about the plaintiff’s win rate among cases that are litigated. The modern literature on this issue began with Priest & Klein (1984), who were the first authors to emphasize the fact that the set of litigated cases is not likely to be a random sample of all disputes. Virtually all subsequent analysis has accepted this important point. More controversially, Priest & Klein (1984, p. 5) argued that “where the gains or losses from litigation are equal to the parties, the individual maximizing decisions of the parties will create a strong bias toward a rate of success for plaintiffs at trial or appellants”.

2By “legal rule”, I mean the full set of rules or standards that determine which party wins in the event of litigation, given the evidence presented at trial.

3For recent examples of studies attempting to study the effects of pleading standard changes due to the Bell Atlantic Corp. v. Twombly, 550 U.S. 554 (2007) and Ashcroft v. Iqbal, 556 U.S. 662 (2009), cases, see, e.g., Moore (2012) and Hubbard (2013). See, e.g., Gelbach (2014) for a selection-based critique of this approach.

4I shall not even attempt a general review the voluminous literature concerning the general properties of litigation models; several excellent recent literature reviews are Spier (2007), Daughety & Reinganum (2012), and Wickelgren (2013).
They further argued that when the defendant’s stakes in litigation are greater than the plaintiff’s, the plaintiff’s win rate in cases observed being litigated would systematically exceed 50 percent, with the opposite result occurring when plaintiff’s stakes are greater; see Priest & Klein (1984, p. 25).

Priest and Klein did not clearly state formal analytical results, and proving such results is complicated by the fact that the plaintiff’s win rate in their model involves a triple integral over an implicitly defined region with complicated properties. Lee & Klerman (2015b) and Lee & Klerman (2015a) have recently used a clever reparameterization of the model to prove a number of results related to various claims or conjectures in Priest & Klein (1984) (including under more general conditions than those described by Priest and Klein). Much of the analysis in these papers is consistent with the basic force of Priest & Klein (1984). However, Klerman & Lee (2014) show that there are conditions under which the observed plaintiff’s win rate will rise whenever the legal rule moves in a pro-plaintiff direction, contradicting Priest and Klein’s suggestions that it may not be possible to draw inferences concerning the state of the law may from the plaintiff’s win rate in litigated cases.

A number of other authors have attempted to analyze the properties of the plaintiff’s win rate in the presence of settlement-induced selection in litigation. A partial list of notable work includes Wittman (1985); Eisenberg (1990); Hylton (1993); Hylton & Lin (2012) and Friedman & Wittman (2007). For more discussion of this literature, see Lee & Klerman (2015b) and Klerman & Lee (2014).

Both Klerman & Lee (2014) and Lee & Klerman (2015b) analyze more than just the original model described by Priest & Klein. For example, Klerman & Lee (2014) analyze models built on asymmetric information, including both Bebchuk’s (1984) screening model and a modification of Reinganum & Wilde’s (1985) signalling model. All of these models—the Priest and Klein model, as well as the screening and signalling models based on asymmetric information—are “structural” in the sense that they involve a complete statement of party beliefs and optimizing behavior. Because the assumptions about party information and optimal behavior are different in these models, they appear qualitatively different. One value of the reduced form approach I take in this paper is that it allows one to map all these models into a common framework that allows a unified treatment of issues related to the plaintiff’s win rate; see section 7, for an example of this translation exercise.

3 The Reduced Form Approach

Let $q_p$ be the plaintiff’s subjective probability that the plaintiff would win if the case were to be litigated. Similarly, let $q_d$ be the subjective probability the defendant places on the same event, i.e., that the plaintiff would win. Any given case’s party beliefs is fully characterized by the pair of beliefs $(q_d, q_p)$. The three functional elements of a reduced form litigation model are then expressed as functions of these beliefs:

1. The first element of a reduced form model is a joint cumulative distribution function of
party beliefs, \( F_{Q_d Q_p} \). This probability distribution may be either discrete, continuous, or a mixture of the two. When this distribution is continuous, I shall write \( f_{Q_d Q_p} \) for its associated joint density. We may thus regard \((Q_d, Q_p)\) as a pair of random variables taking on specific values, \((q_d, q_p)\), at which the distribution’s functions may be evaluated.

2. The second element of reduced form models is the conditional win rate function, \( w \), such that \( w(q_d, q_p) \) is the probability that the plaintiff would win, in the counterfactual circumstance that the case were to be litigated, given the subjective beliefs \((q_d, q_p)\).

3. The third element of a reduced form litigation selection model is the litigation rule. This is a function \( L \) that tells us the probability that a case will be litigated, given the parties’ subjective beliefs. Some litigation models of interest have a binary litigation rule, in the sense that cases with given party beliefs \((q_d, q_p)\) will either all be litigated or all be settled. In such models—which include the Priest-Klein and variants of the Bebchuk screening model—\( L(q_d, q_p) \) equals either 0 or 1 for every case. We can thus define the litigated set as the set of cases that are litigated in such a model; when the litigation rule is binary, \((q_d, q_p)\) is in the litigated set if \( L(q_d, q_p) = 1 \), and not in the litigated set if \( L(q_d, q_p) = 0 \).

One litigation rule of particular interest is the Landes-Posner-Gould, or LPG, litigation rule.\(^5\) The LPG litigation rule plays an important role in the literature, in part because Priest and Klein assumed it in their paper, and in part simply because it is intuitive: it is equivalent to the rule that the parties settle if and only if there is positive surplus from settlement,\(^6\) and they litigate if and only if there is not. To make this idea concrete, we must define the parties’ subjective expected payoffs from litigation.

Let \( c_p \) and \( c_d \) be the plaintiff’s and defendant’s respective costs of litigating, and let \( s_p \) and \( s_d \) be their costs of negotiating a settlement. Finally, let \( J_d \equiv J \) be the defendant’s expected costs when the plaintiff wins, and let \( J_p \equiv \alpha J \) be the plaintiff’s expected benefits in that same event. When \( \alpha = 1 \), we have symmetric stakes, when \( \alpha > 1 \) the plaintiff has greater stakes, and when \( \alpha < 1 \) the defendant has greater stakes. The quantity \( q_p \alpha J - c_p \) is the plaintiff’s expected gain from litigating. The defendant’s expected cost from litigating is \( q_d J + c_d \). The gross surplus available from avoiding litigation is the plaintiff’s expected gain less the defendant’s expected cost: \([q_d J + c_d] - [q_p \alpha J - c_p]\). The net surplus from settlement is positive if this gross surplus exceeds total settlement costs, \( s_p + s_d \). After a little algebra, we arrive at the Landes-Posner-Gould litigation rule’s necessary and sufficient condition for litigation to occur:

\(^5\)The basic idea behind this litigation rule was fleshed out in papers by William Landes, Richard Posner, and John Gould. See Landes (1971), Posner (1973), and Gould (1973).

\(^6\)I adopt the convention that the parties litigate when available settlement surplus exactly equals zero; nothing important turns on this convention.
where \( K \equiv (c_d + c_p - s_d - s_p)/J \) is the share of the defendant’s stakes that are accounted for by the net of total litigation costs over total settlement costs. The LPG litigation rule is embedded in the Priest-Klein model and has also been adopted in numerous other settings.\(^7\)

We can represent the situation in which (1) holds with equality via the simple graph plotted in Figure 1; there I assume that there are symmetric stakes (\( \alpha = 1 \)), and, for concreteness, that \( K = 1/3 \). All cases whose value of \((q_d, q_p)\) lies above this frontier are litigated in the LPG framework, and all those lying below it are settled. I call the upward-
sloped line the LPG litigation frontier, because it is the boundary separating the set of cases that are litigated from the set of those that are settled. Thus under the LPG litigation rule, the litigated set is the set of cases that are above and to the left of the LPG litigation frontier.

Recall that the parameter $K$ measures the importance of litigation costs relative to the value of a judgment to the plaintiff; when this parameter is greater, the litigation frontier shifts up. As the plaintiff’s stakes increase relative to the defendant’s, $\alpha$ increases, which causes the LPG litigation frontier to flatten while also shifting it downward. I provide a more general and detailed analysis of how the LPG litigation frontier varies with the parameters $\alpha$ and $K$ in Appendix A.

4 Balance Conditions and the Win Rate

In this section, I first introduce the concept of balance pairs. I then use this concept to establish general results relating the plaintiff’s win rate to one-half. Each balance pair includes two party beliefs that are related to each other in the following important way.

**Definition 1** (Balance Pair of Party Beliefs).

(i) A balance pair of party beliefs is a pair $(q_d, q_p)$ and $(q_{d2}, q_{p2})$ such that $q_{d2} = 1 - q_{p1}$ and $q_{p2} = 1 - q_{d1}$.

(ii) Each member of this pair is the “balance partner” of the other member.

For example, consider point 1 in Figure 2, where $(q_{d1}, q_{p1}) = (0.2, 0.6)$. This point forms a balance pair with point 2, where $(q_{d2}, q_{p2}) = (0.4, 0.8)$. An important property of balance pair partners is that they either lie on opposite sides of what I shall call the “midline”—the line connecting the top left and bottom right of the graph in Figure 2—or they must both lie on the midline at the same point. Notice further that a point lies below the midline if and only if the sum of party beliefs for that point is less than 1. The following lemma, proved in B states two useful results concerning the geometry of balance pairs and the midline.⁸

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⁸Observe that the midline is the set of points in $(q_d, q_p)$-space given by the equation $q_p = 1 - q_d$.  

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1. The line connecting the two points in any balance pair has slope 1.
2. Two balance pair partners that are not on the midline are equal distances from the midline.

Proof. See Appendix B.

In section 4.1, I describe balance conditions on how the litigation rule, the conditional win rate function, and the joint density of party beliefs vary points in the same balance pair. I prove that when these balance conditions hold, the plaintiff’s win rate among litigated cases will always be one-half. The basic idea is to show that: (i) under the stated balance conditions, the plaintiff’s win rate will be one-half among all litigated cases in each balance pair; (ii) the plaintiff’s win rate among all litigated cases is the average plaintiff’s win rate over all balance pairs with litigated cases; and (iii) the average of one-half is one-half. While the balance conditions are very strong, they can be shown to hold in the model that Priest and Klein simulated with a decision standard of $y^* = 0$. (See section 7.2 below.)
In section 4.2, I then present another theorem, which shows that the plaintiff’s win rate among litigated cases will always differ from one-half if the balance conditions discussed in section 4.1 are violated systematically in one direction or the other.

### 4.1 Sufficient Conditions for the Plaintiff’s Win Rate to Equal $\frac{1}{2}$

My first condition is called simple balance. It involves equality of a function’s value at each partner of a balance pair.

**Definition 2 (Simple Balance).**

A function $h$ defined on $[0, 1] \times [0, 1]$ satisfies *simple balance* with respect to the point $(x, y)$ if and only if $h(x, y) = h(1 - y, 1 - x)$. The function is *globally simply balanced* if and only if it is simply balanced with respect to all points in its domain.

Thus if the litigation rule is simply balanced with respect to given balance pair partners, then each point in the balance pair has the same probability of being litigated. If the joint density of beliefs is simply balanced with respect to a balance pair, then the members of that pair have the same density.

Next consider another kind of balance condition, which will be useful for the conditional win rate function.

**Definition 3 (Complementary Balance).**

The conditional win rate function is *complementarily balanced* with respect to the point $(x, y)$ if and only if $w(x, y) + w(1 - y, 1 - x) = 1$. The function is *globally complementarily balanced* if and only if it is complementarily balanced with respect to all points in its domain.

For example, the conditional win rate function is globally complementarily balanced when it equals the simple mean of the parties’ subjective beliefs: $w(q_d, q_p) = \frac{1}{2}(q_d + q_p)$, and $w(1 - q_p, 1 - q_d) = \frac{1}{2}(2 - q_p - q_d) = 1 - w(q_d, q_p)$.

The following theorem is my first general result. It shows that the balance conditions just defined can be used to form sufficient conditions for the plaintiff’s win rate to equal one-half.
Theorem 1 (Sufficient Conditions for the one-half Result).
The plaintiff’s win rate will equal one-half if the litigation rule satisfies global simple balance, and, with respect to any balance pair whose members have positive probability of being litigated:

(i) the joint distribution of party beliefs is simply balanced, and
(ii) the conditional win rate function satisfies complementary balance.

The formal proof is a bit involved at the full level of generality expressed in the theorem, so I relegate it to Appendix D. But the basic logic can be understood with a special case using the LPG litigation rule, as depicted in Figure 3. In the figure, the line given by $q_p = q_d + K$ is the LPG litigation frontier with symmetric stakes. The corresponding litigation rule is globally simply balanced. Therefore, if a case with beliefs represented by a point is litigated, then so is its balance pair partner.

As in Figure 2 above, points 1 and 2 in this graph form a balance pair, with $(q_{dB}, q_{pB}) = (0.2, 0.6)$ and $(q_{dA}, q_{pA}) = (0.4, 0.8)$, and both are litigated. When the density is simply balanced with respect to these belief points, the average win rate among all points in the population of cases that have these beliefs will equal the simple average of the conditional win rate function values at the two points, i.e., it will equal $\frac{1}{2}[w(q_{dB}, q_{pB}) + (q_{dA}, q_{pA})]$. When
the conditional win rate function is complementarily balanced with respect to the points of a balance pair, it sums to one. Thus the simple average of the conditional win rate function for this balance pair is $\frac{1}{2}$. Since this argument holds for every litigated balance pair, plaintiff’s average win rate is exactly one-half for each litigated balance pair. The average of one-half is one-half, so the average conditional win rate over all litigated cases must be one-half, proving the result in the LPG litigation rule case.

### 4.2 Sufficient Conditions for a Win Rate Other than $\frac{1}{2}$

I now turn to conditions sufficient for us to conclude that the plaintiff’s win rate will be something other than one-half. Consider first the role of stake asymmetry in the Priest-Klein model. Figure 4(a) depicts a situation with greater plaintiff stakes, i.e., $\alpha > 1$. The litigation frontier in this figure lies below the point A, but above its balance pair partner B. Consequently, point A is in the litigated set, but point B is not. Assuming above-midline cases have greater conditional win rates than do their balance pair partners, the effect of greater plaintiff’s stakes is to absorb into the litigated set some cases that plaintiffs are relatively likely to win. This effect will tend to increase the plaintiff’s win rate above its symmetric-stakes level. Figure 4(b) illustrates the opposite situation, in which the defendant’s stakes are greater. Above-midline cases, which have relatively high conditional win rate values, are excluded in this situation. Thus a decision standard above zero tends to reduce the plaintiff’s win rate below its symmetric-stakes level.

I refer to systematic skewing of the litigation rule, or the joint density of party beliefs, in one direction or another relative to the midline as “leaning above” or “leaning below.”

**Definition 4 (Leaning Simply Above or Below the Midline).**

(i) A function $h$ leans simply above the midline if and only if $h(q_d, q_p) > h(1 - q_d, 1 - q_p)$ when $(q_d, q_p)$ lies above the midline.

(ii) The function leans simply below the midline if and only if the inequality in (i) is reversed when $(q_d, q_p)$ lies above the midline.
The second function property will be useful for characterizing imbalance in the conditional win rate function.

**Definition 5 (Leaning Complementarily Above and Below the Midline).**

(i) The conditional win rate function $w$ leans complementarily above the midline at beliefs $(q_d, q_p)$ if and only if $w(q_d, q_p) > 1 - w(1 - q_d, 1 - q_p)$ when $(q_d, q_p)$ lies above the midline.

(ii) The conditional win rate function leans complementarily below the midline at beliefs that lie below the midline if and only if the inequality in (i) is reversed.

The following theorem shows that the two imbalance properties just defined may be used to construct sufficient conditions under which the plaintiff’s win rate will always either exceed one-half, or fall short of one-half.
Theorem 2 (Sufficient Imbalance Conditions for a Win Rate Above or Below one-half). Suppose that the conditional win rate function leans simply above the midline. In addition, for any point above the midline, suppose that at least one of the following inequalities holds strictly, with none being violated:

1. The density leans above the midline.
2. The litigation rule leans above the midline.
3. The conditional win rate function leans complementarily above the midline.

Then the plaintiff’s win rate among litigated cases will exceed one-half. If any leaning is below the midline rather than above, then the plaintiff’s win rate among litigated cases will be less than one-half.

Proof. See Appendix E.

This theorem generalizes the illustrative discussion related to asymmetric stakes, above. It shows that when any leaning of the litigation rule, joint density, or conditional win rate function is in the same direction, the plaintiff’s win rate will be biased in that direction relative to one-half. Note, though, that the theorem does not state that such effects are monotonic with respect to any parameter that controls the extent of leaning; that result appears not to hold.\(^9\)

A natural question is what can be said when there is leaning in both directions: for example, there might be greater plaintiff’s stakes, even as the conditional win rate function leans complementarily below the midline. In such situations, there are effects pushing the plaintiff’s win rate both above and below one-half. Without knowing the magnitudes of such countervailing effects, there will be no way to assess which dominates. Consequently, general ordering relations between one-half and the observed plaintiff’s win rate are impossible unless all imbalances point in the same direction. This fact immediately casts doubt on general claims of the form “asymmetric stakes will cause the plaintiff’s win rate to be greater than one-half.” These claims might be true in some particular circumstances, to be sure. But as the analysis in the next section suggests, they will not hold generally.

\(^9\)For example, Lee & Klerman (2015b) report simulation results indicating that there can be non-monotonicity of the plaintiff’s win rate with respect to the degree of stake asymmetry, other things equal.
5 Any Plaintiff’s Win Rate is Possible—Even Without Asymmetric Information and Regardless of the Stakes

In a classic paper, Shavell (1996) demonstrated that any plaintiff’s win rate between 0 and 1, inclusive, is possible. To prove this result, Shavell used an adaptation of the Bebchuk (1984) screening model, in which the informed side of the dispute can take on two belief types. In my notation, this means that if defendants are the informed side, then the joint distribution places positive probability on the party beliefs \((q_d, q_p) \in \{(q_{d1}, \overline{q}_p), (q_{d2}, \overline{q}_p)\}\), where \(\overline{q}_p\) is the population-weighted average of \(q_{d1}\) and \(q_{d2}\), and, for concreteness, \(q_{d1} < q_{d2}\); in the analogous model with informed plaintiffs, the joint distribution places positive probability on the party beliefs \((q_d, q_p) \in \{(\overline{q}_d, q_{p1}), (\overline{q}_d, q_{p2})\}\), where \(\overline{q}_d\) is the population-weighted average of \(q_{p1}\) and \(q_{p2}\), with \(q_{p1} < q_{p2}\). Shavell shows the following:

- In the informed-defendant model, there is always a separating equilibrium in which the case with \(q_d = q_{d1}\) is litigated and the other case is settled, so that the plaintiff’s win rate is \(q_{d1}\). Further, in such an equilibrium, any \(q_{d1} \in [0, 1)\) is possible.

- In the informed-plaintiff model, there is always a separating equilibrium in which the case with \(q_p = q_{p2}\) is litigated and the other case is settled, so that the plaintiff’s win rate is \(q_{p2}\). Further, in such an equilibrium, any \(q_{p2} \in (0, 1]\) is possible.

- Thus any logically possible plaintiff’s win rate—any rate between \([0, 1]\)—can be an equilibrium when there are two points of support in the population distribution of cases.

In discussing this result, Shavell treats asymmetric information as especially relevant to understanding his anything-goes result. Shavell writes: “Plaintiffs win at trial with a frequency tending toward 50 percent [in the Priest-Klein model] when (1) parties obtain very accurate information about trial outcomes and when (2) the information that each receives is statistically identical.” Page 499. But my Theorem 1 shows that there is nothing necessary about the parties’ having “very accurate information.” What is important in explaining a tendency toward one-half is the right constellation of balance properties elements of the reduced form. It just so happens that as “parties obtain very accurate information about trial outcomes” in the Priest-Klein model, these balance conditions are achieved when the stakes are symmetric.\(^{10}\) Elaborating on this idea, Shavell notes that while party beliefs “differ from each other by chance in particular instances,” page 499, the assumption of statistically identical information “implies that the distribution of plaintiff beliefs about victory is essentially the same as the distribution of defendant beliefs about plaintiff victory.” Page 499.

\(^{10}\) I prove this fact in a companion paper, XXX.
But using the reduced form, it is straightforward to show that the observed plaintiff’s win rate can take on any value even without asymmetric information of the form Shavell assumes. Further, this is true even when the parties have the same marginal belief distributions. Thus, neither of the features of the PK model that Shavell identifies as driving the one-half result in fact explains it. As Shavell did, I shall show my results using a very simple constructive example, with a discrete distribution of case types. Suppose:

1. The litigation rule is the LPG litigation rule with parameter $K \in (0, 1)$ and stakes $\alpha$, so that cases are litigated if and only if $q_p > \alpha^{-1}(q_d + K)$. Further, assume that $\alpha \in (K, 1 + K)$, so that a case with $(q_d, q_p) = (0, 1)$ will be litigated.

2. The conditional win rate function is a weighted average of parties’ subjective beliefs: $w(q_d, q_p) = \theta q_d + (1 - \theta)q_p$, where $\theta \in [0, 1]$.

3. The joint distribution of party beliefs is given by the discrete probability function

$$f_{Q_dQ_p}(q_d, q_p) = \begin{cases} 
\rho, & \text{if } (q_d, q_p) = (q_{dL}, q_{pL}) \\
\rho, & \text{if } (q_d, q_p) = (q_{pL}, q_{dL}) \\
\gamma - \rho, & \text{if } (q_d, q_p) = (1, 1) \\
1 - \gamma - \rho, & \text{if } (q_d, q_p) = (0, 0),
\end{cases}$$

where $q_{pL} > \alpha^{-1}(q_{dL} + K)$ and $\gamma \in (0, 1)$.

Notice that the final inequality means that the first belief point, $(q_{dL}, q_{pL})$, is litigated. Notice as well that the second belief point, $(q_{pL}, q_{dL})$, is not the first point’s balance pair partner; rather, I have constructed the second belief point by interchanging the parties’ beliefs at the first belief point. It can be shown that the second belief point will never be in the litigated set. It follows that the plaintiff’s win rate is $W^L(q_{dL}, q_{pL}; \theta) \equiv \theta q_{dL} + (1 - \theta)q_{pL}$ for all positive values of $\rho$. By varying $\theta$ between 0 and 1, we can vary this value from $q_{pL}$ to $q_{dL}$. Since the point $(q_{dL}, q_{pL}) = (0, 1)$ is always in the litigated set, we can choose $\theta$ and $(q_{dL}, q_{pL})$ to achieve any desired value of $W^L(q_{dL}, q_{pL}; \theta) \in [0, 1]$. This proves that any plaintiff’s win rate is possible in this model, even with the LPG litigation rule, and without asymmetric information of the form Shavell assumed.\(^{12}\)

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\(^{11}\)This claim is equivalent to $\alpha q_{dL} < q_{pL} + K$, or $q_{pL} > \alpha q_{dL} - K$. I shall prove that the right hand side of this inequality is less than $\alpha^{-1}(q_{dL} + K)$. This is clearly true when $\alpha \leq 1$. When $\alpha > 1$, it is true if and only if $\alpha^{-1}(q_{dL} + K) > \alpha q_{dL} - K$, which implies $(\alpha^2 - 1)q_{dL} < (1 + \alpha)K$, which holds if and only if $(\alpha - 1)q_{dL} < K$. Since $\alpha < 1 + K$ by hypothesis, the left hand side is less than $K q_{dL} \leq K$, since $q_{dL} \leq 1$. Therefore, $\alpha q_{dL} - K < \alpha^{-1}(q_{dL} + K) < q_{pL}$, with the second inequality holding since $(q_{dL}, q_{pL})$ is litigated. We have thus established that $q_{dL} < \alpha^{-1}[q_{pL} + K]$, which is necessary and sufficient condition for all cases with $(q_d, q_p) = (q_{pL}, q_{dL})$ to be litigated, given the LPG litigation rule.

\(^{12}\)One might argue that there is asymmetric information here in the sense that when $\theta \neq \frac{1}{2}$, one party’s belief is systematically more accurate than the other’s. While this is true, there is still an important
Finally, observe that in the present example, the parties have identical marginal belief distributions. This is true since each party has probability $\rho$ of believing the plaintiff’s chance of victory is $q_{pL}$, probability $\rho$ of believing the plaintiff’s chance of victory is $q_{dL}$, probability $\gamma - \rho$ of believing the plaintiff is certain to win, and probability $1 - \gamma - \rho$ of believing the plaintiff is certain to lose. Thus, even when the parties have identical belief distributions, any plaintiff’s win rate is possible.

This result can be pushed further, to show that Shavell was incorrect in suggesting that sufficient conditions for the plaintiff’s win rate to converge to one-half are (i) for the party belief distributions to be identical and (ii) for parties’ beliefs to converge to perfect accuracy. For any $\rho > 0$, the plaintiff’s win rate among litigated cases will be $W^L(q_{dL}, q_{pL}; \theta)$. Consider a sequence of $\rho$ values that converges to 0. Since $W^L(q_{dL}, q_{pL}; \theta)$ does not depend on $\rho$, the plaintiff’s win rate among litigated cases must be constant along this sequence. Since the limit of a constant is that same constant, the limiting value of the plaintiff’s win rate as $\rho$ goes to zero is, again, $W^L(q_{dL}, q_{pL}; \theta)$—and we have seen that this function can take on any value in $[0,1]$. Thus even as both parties’ information becomes perfect, it is possible to observe any plaintiff’s win rate in $[0,1]$ among litigated cases. What makes this surprising result possible is that, even though at least one party will be mistaken in all litigated cases, cases for which the parties’ beliefs are accurate will not be litigated.\(^\text{13}\)

Further, my argument above proceeded with a fixed value of $\alpha$ anywhere in the interval $(K, 1+K)$. Thus nothing in this argument involves any condition on symmetry or asymmetry of the stakes. Thus there is also no necessary link between the plaintiff’s win rate and either symmetry/asymmetry of stakes or party information.

Finally, the analysis in this section shows that Priest & Klein (1984) were incorrect to suggest that the nonrandom selection of cases for litigation causes at least a bias in the plaintiff’s win rate toward one-half. It is enough to consider again the example used above to prove that any plaintiff’s win rate is possible. A bit of algebra shows that in that example, if all cases were litigated the plaintiff’s win rate would be

$$W^{ALL} \equiv \gamma - \rho[1 - (q_{dL} + q_{pL})]. \quad (2)$$

\(^\text{13}\)To show the result precisely, observe that as $\rho$ goes to 0, only cases with $(q_{dL}, q_{pL}) = (0,0)$ or $(q_{dL}, q_{pL}) = (1,1)$ occur with positive probability. For each of these types of cases, since $W^L(q_{dL}, q_{pL}; \theta)$ is a $\theta$-weighted average of party beliefs, the true plaintiff’s conditional win rate equals the parties’ subjective beliefs concerning this rate: $W^L(q_{dL}, q_{pL}; \theta) = q_{dL} = q_{pL}$. Therefore as $\rho \to 0$, the parties make virtually no mistakes in the overall population of cases—yet the plaintiff’s win rate among the ever-shrinking set of litigated cases remains $W^L(q_{dL}, q_{pL}; \theta)$.\(^\text{15}\)
Now let \((q_{dL}, q_{pL})\) be any point on the midline inside the litigated set, so that \(q_{dL}+q_{pL} = 1\). Then the population plaintiff’s win rate is \(W^{\text{ALL}} = \gamma\). Since \(\gamma\) does not affect the plaintiff’s win rate among litigated cases, \(W^L(q_{dL}, q_{pL}; \theta)\), it will always be possible to choose values of \(q_{dL}, q_{pL}, \theta\), and \(\gamma\) such that either \(W^L(q_{dL}, q_{pL}; \theta) > W^{\text{ALL}} > 0.5\) or \(W^L(q_{dL}, q_{pL}; \theta) < W^{\text{ALL}} < 0.5\). Thus, it is entirely possible for the selection of disputes for litigation to cause a bias away from one-half, contrary to Priest & Klein’s (1984) argument.\(^{14}\)

6 Changes in the Plaintiff’s Win Rate as the Decision Standard Changes

Recently, Klerman & Lee (2014) (“KL”) have argued that changes in the win rate usefully indicate the direction of changes in the decision standard following statutory or doctrinal innovations. KL provide both extensive simulations of the original Priest-Klein model, whose results indicate that a pro-plaintiff shift in the decision standard induce an increase in the plaintiff’s win rate, and an analytical proof that the result holds under what KL treat as weak conditions. KL also consider asymmetric-information models of litigation, providing conditions under which a pro-plaintiff change in the decision standard will necessarily cause the plaintiff’s win rate to rise; they appear to believe these conditions are also relatively weak.

KL’s observation that we often care about the direction of change in legal rules, and not just their level, is an astute one. And the range of evidence they offer is impressive. This is especially true in the context of a literature that has had its share of believers in some version of the hypothesis that the plaintiff’s win rate can always be expected to be near one-half regardless of the actual legal rules in place. However, their argument for using changes in the win rate to measure the direction of change in legal rules should be rejected. To see why, it is enough to provide a counterexample to the KL result using my reduced form approach. (I leave the precise numbers to Appendix F, focusing here on an intuitive understanding of selection patterns following a change in legal rules.)

When the decision standard moves in a pro-plaintiff direction, we can expect the conditional win rate function to shift in favor of plaintiffs. That is, holding constant the parties’ subjective beliefs about the plaintiff’s win probability, an across-the-board pro-plaintiff change in the decision standard can be expected to increase the actual probability that the plaintiff will win any given case. A weaker version that is still fairly described as “pro-plaintiff” would (i) increase the conditional win rate function’s value for at least some pair of party beliefs, while (ii) not reducing the conditional win rate for any pair. It is fair to say that any definition of a pro-plaintiff change in the decision standard must at least satisfy this weaker version. I shall refer to the effect just described as the “no-selection effect” of a pro-plaintiff change in the legal rule.

\(^{14}\)The argument used here can also be used to show that it is possible for the plaintiff’s win rates among the full population of cases and the subset of litigated cases to be on opposites sides of one-half.
An additional effect of a pro-plaintiff change in the legal rule is that parties in at least some cases will increase the subjective probability they place on the event that the plaintiff would win in the event of trial—and that no party in any case will reduce this probability. In terms of the graphs I have used so far, then, the effect of such a change in beliefs will be to move party beliefs in the northeast direction.

Consider Figure 5, which depicts a situation in which there are initially equal numbers of each of four types of cases, labeled A, B, C, and D. Following a change in the legal rule that is perceived to be pro-plaintiff, the parties’ beliefs in these cases respectively shift to points $A'$, $B'$, $C'$, and $D'$. Each of the “prime” points lies to the northeast of the initial belief points, reflecting that the parties believe the change in the legal rule increases the plaintiff’s chances of winning in each case.

The upward sloping line in the figure represents the litigation frontier under the LPG litigation rule. Notice that the litigation status cases of types A and B does not change when the legal rule changes. Type A cases are in the litigated set, while type B cases are not. By contrast, the change in the legal rule alters the litigation status of type C and type D cases. Type C cases are litigated under the initial legal rule but are settled under the new rule; thus they are “selected out” of litigation by the rule change. Type D cases exhibit the opposite pattern: they are settled under the initial legal rule and litigated under the new one; type D cases are “selected in” to litigation.

The selection-out phenomenon will cause the plaintiff’s win rate to fall, assuming that type C cases have a greater plaintiff’s win rate than type A cases under the initial rule. The selection-out phenomenon will also cause the plaintiff’s win rate to fall, assuming that Type D cases have a lower plaintiff’s win rate than type A cases under the initial rule. In this example, then, both forms of case selection induced by the change in the legal rule push against the no-selection effect. If the case selection effects are substantial enough, then it is possible that they will outweigh the no-selection effect. The ultimate question is whether the no-selection effect is strong enough to outweigh the combined removal of cases that plaintiffs frequently would win under the initial legal rule, together with the influx of cases that plaintiffs frequently would lose under the initial rule.

This analysis indicates that when a change in legal rules occurs, the plaintiff’s win rate among litigated cases can change in any direction, regardless of the direction in which the decision standard changes.$^{15}$

7 Translating Models in the Literature

To illustrate the flexibility of the reduced form approach, I now show how it encompasses three important structural models in the literature. I start with one-sided asymmetric information models developed by Bebchuk (1984) and Lee & Klerman (2014, “LK”). Then I turn to the model that Priest & Klein (1984) simulated.

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$^{15}$I provide a full constructive example in Appendix F.
Figure 5: Selection Effects and How a Pro-Plaintiff Shift in the Legal Rule Affects the Win Rate
7.1 The Asymmetric Information Models

In both the Bebchuk and LK asymmetric information models, one of the two parties knows the true probability, $\pi$, that the plaintiff would win in litigation. The other party does not know $\pi$, but this uninformed party does know the correct population distribution of $\pi$, $f_\Pi$. The uninformed party’s belief concerning the plaintiff’s probability of winning is thus the population average value of $\pi$: $\bar{\pi} \equiv \int_0^1 \pi f_\Pi(\pi) d\pi$; notice that this means that all uninformed parties have the same subjective belief $\bar{\pi}$.

The joint distribution of beliefs in these models may be written

$$f_{Q_dQ_p}(q_d, q_p) = \begin{cases} f_\pi(q_d), & \text{if } q_p = \bar{\pi} \text{ and defendants are informed}, \\ f_\pi(q_p), & \text{if } q_d = \bar{\pi} \text{ and plaintiffs are informed}, \\ 0, & \text{otherwise}. \end{cases}$$

The conditional win rate function is

$$w(q_d, q_p) = \begin{cases} q_d, & \text{if defendants are informed}, \\ q_p, & \text{if plaintiffs are informed}. \end{cases}$$

Where the Bebchuk screening model and the LK signalling model part ways is in the litigation rule. In the Bebchuk screening model, the uninformed party makes a settlement demand or offer. If the informed party accepts, the case settles, and if the informed party rejects, the case is litigated (thus the possibility of counteroffers is ruled out by assumption). With settlement costs assumed to be zero, it can be shown that the optimal settlement offer by an uninformed defendant, $x^*_d$, depends on the shape of the density $f_\pi$, the ratio $K = (c_p + c_d)/J$, and the defendant’s cost parameter $c_d$. The optimal settlement acceptance rule for informed plaintiffs is to accept when the plaintiff’s probability of winning is less than $q^*_p \equiv (x^*_d + c_p)/(\alpha J)$. When plaintiffs, rather than defendants, are uninformed, the optimal settlement demand $x^*_p$ depends on the shape of $f_\pi$, on $K$, and on the plaintiff’s cost parameter $c_p$, while the informed defendant will agree to the demand whenever the defendant’s belief about the plaintiff’s probability of winning exceeds the threshold $q^*_d \equiv (x^*_p - c_d)/J$. Thus in the Bebchuk model, we have the litigation rule given by

$$L(q_d, q_p) = \begin{cases} 1, & \text{if defendants are informed and } q_d < q^*_d, \\ 1, & \text{if plaintiffs are informed and } q_p > q^*_p, \\ 0, & \text{otherwise}. \end{cases}$$

Notice that this litigation rule is binary. In addition, it depends on the beliefs of only the informed party. There will be either a vertical litigation frontier (informed defendants)
or a horizontal litigation frontier (informed plaintiffs) in \((q_d, q_p)\)-space, with all cases to the left or above the frontier litigated and all others settled.

Now consider the Klerman & Lee (2014) ("KL") signalling model.\(^\text{16}\) The joint density of party beliefs and the conditional win rate function for this model have already been defined, so only the litigation rule remains to be characterized.\(^\text{17}\) The LK signalling model differs from Bebchuk’s in that it is the informed party who makes the settlement offer or demand. For purposes of discussion, I follow Klerman & Lee (2014) and assume symmetric stakes and zero settlement costs \((\alpha = 1 \text{ and } K = (c_p + c_d)/J)\). In the unique class of separating equilibria with informed plaintiffs,\(^\text{18}\) KL’s results imply that the probability a case is litigated is

\[
L(q_d, q_p) = 1 - \exp \left( -\frac{q_p - \bar{\pi}}{K} \right),
\]

where \(\bar{\pi}\) is the lowest value of \(\pi\) in the support of \(f_\pi\). For \(q_p = \bar{\pi}\), the case has zero probability of being litigated, while for any other plaintiff’s belief, the case has interior probability of being litigated. Thus the litigation rule is not binary in the LK signalling model.

It can be shown that when the defendant is the informed party, the analysis is similar, but with

\(^\text{16}\)This model is an adaptation of the Reinganum & Wilde (1985) signaling model. In the Reinganum and Wilde model, both parties in each case know the true probability that the plaintiff would win, but there is asymmetric information concerning the stakes: the informed side knows the true stakes, while the uninformed side knows only the distribution of stakes. Since the focus in both KL and here is on win rates (RW were interested in other issues), it is necessary to allow variation in the probability the plaintiff would win. I follow KL in assuming there is no variation in stakes.

\(^\text{17}\)There is a subtlety involved in defining the uninformed party’s subjective belief, because in a fully separating equilibrium such as the one that KL describe, the uninformed party can use the informed party’s settlement proposal to identify the informed party’s type. Thus, the uninformed party’s “ex post” belief—her belief after the informed party’s settlement proposal is known to both parties—must be the case’s type, which is the informed party’s subjective belief, too. My focus will not be on the uninformed party’s ex post belief, but rather on her ex ante belief, before she knows the informed party’s offer or demand. This is the proper focus for present purposes since what is interesting is to understand how to characterize the ex ante informational structure; in a fully separating equilibrium, the party that is “uninformed” ex ante is always equally informed ex post as the “informed” party, so ex post there is no asymmetric information left to characterize.

\(^\text{18}\)In each separating equilibrium, a plaintiff with knowledge that the case has type \(\pi\) will demand a settlement amount of \(s(\pi) \equiv \pi J + c_d\) for any type \(\pi\) that has positive density. This involves separating since the values of both \(c_d\) and \(J\) are common knowledge among the parties, so that the type of any defendant playing this strategy can be identified by subtracting \(c_d\) from the settlement offer and then dividing by \(J\). In equilibrium, a demand of \(S\) is rejected by the defendant with probability \(r^*(S) = 1 - \exp-(S - \bar{S})/C\) when \(S \in [\pi J + c_d, J + c_d]\) (where \(\pi\) is the lowest case type with positive density); with probability \(r^*(S) = 1\) when \(S > J + c_d\), and with probability \(r^*(S) = 0\) when \(S < \pi J + c_d\). Given the plaintiff’s settlement demand, this means that the litigation probability is interior to \([0, 1]\) except on type sets of measure zero. As KL note, there are multiple sets of defendant beliefs that would support a separating equilibrium, but the settlement offers and rejection probabilities associated with them are unique.
so that there is probability 0 of litigation when \( q_d = 1 \) but otherwise there is interior probability of litigation.

It is easy to see, from simple inspection, that the balance conditions introduced in section 4 provide little purchase on the asymmetric information models just translated. For example, the litigation rule just derived for the signalling model depends on only one party’s belief, and it is monotonic. These facts imply that it will not be globally simply balanced. In both the screening and signalling models, \( w(q_d, q_p) \) equals one of the party’s beliefs and is invariant to the other party’s. This means that for cases with positive density in the overall population of cases, balance pair partners generally will have zero density, i.e. will not exist. Consequently there cannot be balance in the litigation rule or the joint density of beliefs, and the conditional win rate function will not be complementarily balanced. Finally, in these models the uninformed party always has the same belief about the plaintiff’s chance of winning, so that all cases lie on either a horizontal line (informed defendant) or a vertical line (informed plaintiff) in \( q_d - q_p \) space. These observations help clarify why there has never been any claim in the literature that the plaintiff’s win rate tends toward one-half in these models.

### 7.2 The Simulated Model in Priest and Klein (1984)

I shall now show that just the opposite is true of the model that Priest and Klein’s simulated: it tends toward balance in some important cases that have claimed much attention.

In the PK model, there is such a thing as true case quality. It is represented by a random variable \( Y \). The plaintiff would actually win a case if it were litigated any time that true case quality at least meets the decision standard \( y^* \), i.e., when \( Y \geq y^* \); the defendant would win otherwise. Each party receives a signal of true case quality. The plaintiff’s signal equals \( Y_p = Y + \epsilon_p \), and the defendant’s equals \( Y_d = Y + \epsilon_d \). The random variable \( (\epsilon_d, \epsilon_p, Y) \) is normally distributed across the population with mean 0 and variance

\[
\Sigma \equiv \begin{bmatrix}
\sigma^2 & 0 & 0 \\
0 & \sigma^2 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

Priest and Klein assume that, conditional on the event that the parties’ signals are \( Y_p = y_p \) and \( Y_d = y_d \), their subjective beliefs are

\[L(q_d, q) = 1 - \exp \left( -\frac{1 - q_d}{K} \right)\]

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19Lee & Klerman (2015b) consider variations on the PK model that weaken the joint-normality assumption on the random vector \( (\epsilon_p, \epsilon_d, Y) \). For simplicity, I stick to the concrete assumption that Priest and Klein make.
\[ q_p = \Phi \left( \frac{y_p - y^*}{\sigma} \right) \quad \text{and} \quad q_d = \Phi \left( \frac{y_d - y^*}{\sigma} \right), \quad (3) \]

where \( \Phi \) is the cdf of the standard normal distribution.\(^{20}\) As noted above, Priest and Klein assume that cases are litigated if and only if it is impossible for both parties to gain from a settlement, i.e., they adopt the LPG litigation rule: 

\[ L(q_d, q_p) = 1 \text{ if } \alpha q_p > q_d + K, \text{ and } L(q_d, q_p) = 0 \text{ otherwise.} \]

Because of the presence of true case quality in both the parties’ signals, the random variables \( Y_p \) and \( Y_d \)—and thus the parties’ subjective beliefs—will be positively dependent across cases, even though \( \epsilon_p \) and \( \epsilon_d \) are independent. I show in Appendix G, the joint density of party beliefs in the simulated PK model may be written

\[ f_{Q_d Q_p}(q_d, q_p) = A(y^*, \sigma) \exp \left[ \frac{-1}{2\sigma^2(2 + \sigma^2)} \left( \left[ \Phi^{-1}(q_d) - \Phi^{-1}(q_p) \right]^2 - 2\Phi^{-1}(q_d)\Phi^{-1}(q_p) + 2y^*\sigma(1 + \sigma^2)\left[ \Phi^{-1}(q_d) + \Phi^{-1}(q_p) \right] \right) \right], \quad (4) \]

where the function \( A \) does not depend on party beliefs. I also show in Appendix G that the conditional win rate function in the model that Priest and Klein simulated is

\[ w_{PK}(q_d, q_p) = \Phi \left( \frac{\Phi^{-1}(q_p) + \Phi^{-1}(q_d) - \sigma y^*}{\sqrt{2 + \sigma^2}} \right). \quad (5) \]

The functional forms of the joint density and the conditional win rate function have some special features. First, observe that when \( y^* = 0 \), the conditional win rate function has the form

\[ w_{PK0}(q_d, q_p) = \Phi \left( \frac{\Phi^{-1}(q_p) + \Phi^{-1}(q_d)}{\sqrt{2 + \sigma^2}} \right). \quad (6) \]

It is tedious but straightforward to show that this function is globally complementarily balanced. Similarly, the joint density \( f_{Q_d Q_p} \) is globally simply balanced when \( y^* = 0 \). Finally, with symmetric stakes (\( \alpha = 1 \)), we have seen that the LPG litigation rule adopted in the PK model is also globally simply balanced. By Theorem 1, then, the plaintiff’s win rate must be one-half when the decision standard is \( y^* = 0 \) and the stakes are symmetric.

Next, observe that \( y^* \) enters the conditional win rate function only as part of the term \( \sigma y^* \). The same is true of the exponential part of \( f_{Q_d Q_p} \). Thus even with a decision standard quite far from zero, the conditional win rate function and the joint density will be approximately

\(^{20}\)It can be shown that these beliefs are inconsistent with the information embodied in the signals \( Y_p \) and \( Y_d \) (see Lee & Klerman (2015a) for a discussion). The beliefs embodied in (3) can still be part of a reduced form selection model, however, and I shall take them as given.
balanced in the “right” ways if $\sigma$ is close enough to zero.\footnote{In calculating the plaintiff’s win rate, the multiplicative factor $A(y^*, \sigma)$ winds up in both a numerator and a denominator, so that it cancels and may be ignored for $\sigma > 0$. That said, as $\sigma$ goes to 0, this factor converges to zero, which makes the necessary limiting argument technically complicated due to the presence of $A(\cdot)$ in the denominator. But the basic logic of treating “small” $\sigma$ the same as $\sigma$ converging to 0 does go through. I address these issues in a companion paper, XXX; see also Lee & Klerman (2015b), who confront the same problem from a mathematically different angle.}

This explains why the plaintiff’s win rate in the simulated Priest-Klein model appears to converge to one-half as $\sigma$ becomes closer to zero, given symmetric stakes. As $\sigma$ becomes ever smaller, the conditional win rate function and joint density of party beliefs in this model move ever closer to global simple balance. Conversely, values of $\sigma$ far from zero, coupled with a non-zero value of the decision standard, yield a plaintiff’s win rate farther from one-half.

In sum, when the stakes are symmetric, increased accuracy of party information pushes the key elements of the reduced form of the Priest-Klein model toward the relevant forms of global balance. That explains why increased accuracy of party information in the Priest-Klein model is associated with a plaintiff’s win rate close to one-half.\footnote{As noted above, Lee & Klerman (2015b) prove analytically that with symmetric stakes, the plaintiff’s win rate converges to one-half as $\sigma$ converges to zero, in a considerably more general version of the Priest-Klein model than the one Priest and Klein simulated. In a companion paper, XXX, I show that Lee and Klerman’s result may also be regarded as the result of convergence of the conditional win rate and joint density functions toward the right kinds of balance.}

8 Conclusion

In this paper I have provided a new and general approach to understanding selection in litigation, which is centered on three elements: the conditional win rate function, the joint density of party beliefs, and the litigation rule. My reduced form approach is highly fruitful. For example, since Priest & Klein’s (1984) seminal contribution, much argument has centered around whether the plaintiff’s win rate can be expected to be close to one-half, and what factors will tend to drive it away from that value. My approach allows one to use simple, interpretable “balance” and “imbalance” conditions to answer these questions. And the answers are interesting: there is no general tendency of the plaintiff’s win rate toward one-half, even when stakes are symmetric, even when the parties have identical belief distributions, and even when party information is very good.

Among other things, this means limiting results such as those conjectured by Priest & Klein (1984) and proved in Lee & Klerman (2015b) are special to the structural contexts posited. Further, my reduced form approach allows one to construct simple examples contradicting the suggestion in Klerman & Lee (2014) that the plaintiff’s win rate can be expected to move predictably when legal rules change in favor of one side of the $v$.

Further, I conjecture that the reduced form approach will help us understand how seemingly very different structural litigation models—such as the screening, signalling, and Priest-Klein models discussed here—are related to each other, but that topic is for another day.
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A Analysis of the LPG Litigation Frontier

Figure 6 illustrates how the phenomenon of asymmetric stakes affects the litigation frontier, and thus the litigated set. Each solid line in this figure plots the LPG litigation frontier for a different value of \( \alpha \), i.e., each line plots \( q_p = \alpha^{-1}(q_d + K) \) for some pair of \( \alpha \) and \( K \) values. With one exception, I set \( K = \frac{1}{3} \) in all plots in the figure. The arrows pointing left from the various litigation frontiers indicate that the litigated set consists of all cases represented by pairs of parties’ subjective beliefs that lie above and to the left of the frontier. The top line is the litigation frontier for which \( \alpha = 0.5 \), so that the defendant’s stakes are twice the plaintiff’s stakes (recall that \( J_p = \alpha J_d \)). The second line down is the litigation frontier for which \( \alpha = 1 \), which is the symmetric stakes case; note that the slope of this litigation frontier is 1. The third line down is the litigation frontier for which \( \alpha = 1.3 \). Since \( \alpha > 1 \), the plaintiff’s stakes are greater in this situation; notice that this litigation frontier does not cross the 45-degree line.

The next solid line is the litigation frontier for which \( \alpha = 2 \). This line crosses the 45-degree line and never makes it to a point where \( q_p = 1 \). When the plaintiff’s stakes are this much greater than the defendant’s, there are subjective plaintiff’s beliefs great enough such that the parties will never settle, even when the defendant is sure the plaintiff will win. This is evident since the litigation frontier with \( \alpha = 2 \) intersects the point where \( q_d = 1 \) at a value of \( q_p \) less than one. It is possible to show that the LPG litigation frontier intersects the 45-degree line only if \( \alpha \geq 1 + K \).23 Finally, the lowest line in Figure 1(b) is plotted with \( \alpha = 2 \) and \( K = -0.25 < 0 \). In this situation, settlement costs are sufficiently greater than litigation costs that the parties litigate even when both are certain that the plaintiff

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23This situation is important in Lee & Klerman’s (2015b) analysis of a generalization of the Priest-Klein model.
will lose: even cases with beliefs \((q_d, q_p) = (0, 0)\) are litigated in this situation. I note that there is reason to assume that such a situation would never happen in practice, as long as plaintiffs have the option to simply drop their suits. It is not credible for the plaintiff to litigate in this situation, since litigation is costly and, with \(q_p = 0\), entirely without any benefit to the plaintiff. For simplicity, I ignore credibility issues in this paper, but virtually all my qualitative results would hold even with credibility imposed.\(^{24}\)

Notice that as the defendant’s stakes increase relative to the plaintiff’s—i.e., as \(\alpha\) falls—both the slope and the intercept of the LPG litigation frontier increase: the LPG litigation frontier moves toward the upper-left corner, so that, all else equal, ever fewer cases will be litigated. When \(\alpha < K\), no cases are litigated, because then the litigation frontier intersects the vertical axis at a point where \(q_p > 1\); this means settlement surplus is always positive because the stakes are so high for the defendant, and all cases will be settled. When \(\alpha > K\) and \(K\) is held constant, both the slope and the intercept of the LPG litigation frontier fall as the plaintiff’s relative stakes level grows (i.e., as \(\alpha\) grows). Thus with \(K\) held constant, increases in \(\alpha\) expand the set of cases that could be litigated, while reductions in \(\alpha\) cause this set to shrink. In sum, relatively greater plaintiff’s stakes are associated with more litigation, and vice-versa.

### B Proof of Lemma 1

**Lemma 1** (The Geometry of Balance Pairs and the Midline).

1. The line connecting the two points in any balance pair has slope 1.
2. Two balance pair partners that are not on the midline are equal distances from the midline.

*Proof.* The first part holds because the height and width between the points is the same. The height is given by the length of the line connecting points 1 and 3 in Figure 2, is \(q_{p2} - q_{p1} = 1 - q_{d1} - q_{p1}\). The width between points 2 and 4, given by the length of the line connecting points 3 and 2, which is \(q_{d2} - q_{d1} = 1 - q_{p1} - q_{d1}\), establishing the first claim. To prove the second claim, observe that in terms of Figure 2 it means the length of the line connecting points 1 and 4 and the line connecting points 2 and 4 must be the same. To prove this, consider two triangles: the one formed by the line connecting points 1, 3 and 4, and the one formed by the line connecting points 2, 3 and 4. We have seen that the lengths between 1 and 3 and between 2 and 3 are the same. The segment of the midline connecting 3 and 4 is common to the two triangles. By construction the line connecting 1 and 3 is perpendicular to the line connecting 2 and 3, so their angle of intersections must be 90 degrees. The midline has slope -1, so the two angles it makes between itself and the 1–3 and 3–2 sides each must be 45 degrees. Thus the triangles in question have two sides with

\(^{24}\)This is true because the effect of imposing the credibility constraint can be shown to be functionally equivalent to making the litigation frontier be the max of the horizontal line at \(q_p = c_p/(\alpha J)\) and the LPG litigation frontier.
the same length, with the angle between these sides being the same in the two triangles. This “side-angle-side” equivalence is sufficient for the two triangles to be congruent. Since corresponding parts of congruent triangles are congruent, it follows that the line connecting points 1 and 4 and the line connecting points 4 and 2 must have equal length, establishing the equidistance claim.

\[
C \quad \text{Asymmetric Stakes and the Bifurcation of the Litigated Set with the LPG Litigation Rule}
\]

It will help to define some properties that can be used to characterize the litigated sets under the LPG litigation rule.

**Definition 6 (Balance and Leaning Properties of Sets).**

1. A set \( X \) is *balanced* if and only if either both partners of any balance pair are litigated, or neither is: \((q_d, q_p) \in X\) implies that \((1 - q_p, 1 - q_d) \in X\).

2. A set \( X \) *leans above* the midline if and only if there exists at least one above-midline point \((q_{dA}, q_{pA}) \in X\) such that (i) its balance pair partner is not in \(X\), i.e., \((1 - q_{pA}, 1 - q_{dA}) \notin X\), and (ii) there does not also exist a below-midline point \((q_{dB}, q_{pB}) \in X\) whose balance pair partner is not in \(X\).

3. A set \( X \) *leans above* the midline if and only if there exists at least one below-midline point \((q_{dB}, q_{pB}) \in X\) such that (i) its balance pair partner is not in \(X\) and (ii) there does not also exist an above-midline point \((q_{dA}, q_{pA}) \in X\) whose balance pair partner is not in \(X\).

The following lemma shows that under the LPG litigation rule, the balance and leaning properties of the litigated set are directly connected to stakes asymmetry and symmetry.

**Lemma 3 (Characterizing the LPG litigation rule and leaning).**
Suppose the LPG litigation rule holds. Then:

1. If the stakes are symmetric, the litigation rule is balanced.

2. If the plaintiff’s stakes are greater, then the litigated set leans above the midline.

3. If the defendant’s stakes are greater, then the litigated set leans below the midline.
Figure 7: Properties of the Litigated Set Under the LPG Litigation Rule

(a) $\alpha = 1.4$, $K = 0.6$

(b) $\alpha = 0.6$, $K = 0.3$

Proof. A simple way to prove part 1, is to recall that the line connecting any two balance pair partners must have slope 1 in $(q_d, q_p)$-space. Consequently, this line cannot cross the LPG litigation frontier when the stakes are symmetric, since the frontier has slope 1 in that event. Thus both members of a balance pair must lie on the same side of the litigation frontier, which means that either both balance pair partners are in the litigated set or neither are. That is the definition of a balanced set.

A formal algebraic proof of the claims related to asymmetric stakes is straightforward, but a simple graphical illustration is more illuminating. Consider Figure 7(a). The line that connects points 1, 4, and 5 is the litigation frontier, drawn here to have greater plaintiff’s stakes, so that it has slope less than one. Under the LPG litigation rule, all cases above and to the left of this frontier are litigated, so the litigated set is the union of the two shaded regions given by triangle 3-4-5 and polygon 1-2-3-4. By construction, triangle 3-4-5 lies entirely above the midline, so the set of points in it leans above the midline. Meanwhile, the set of points in polygon 1-2-3-4 can be shown to be balanced.\(^{25}\) Thus the litigated set in this

\(^{25}\)By the same logic used to prove part 1 of the Proposition, it follows that the triangle of points with vertices at points 1, 2, and 3 satisfy simple balance. The line connecting points 1 and 3 has equation $q_p = q_d + K/\alpha$, so it has slope 1. Now suppose that we know that the points on the lines 1-3 and 1-4 linked by lines of slope 1 are equidistant from the midline. Then it would follow that points A and B are balance pair partners, from which it would immediately follow that the balance pair partner of any below-midline point inside triangle 1-3-4 must also lie inside the triangle (since balance pair partners are always equidistant from the midline and lie on a line having slope 1). Thus it is enough to show that any point B on line 1-4 is the balance pair partner of a point such as A on line 3-4. This result can be proved by plugging the balance pair partner beliefs into the LPG litigation rule, which yields the equation $1 - q_d = \alpha^{-1}(1 - q_p + K)$, or $q_p = \alpha q_d + 1 + K - \alpha$. This is the equation for the line connecting points 3 and 4 (note that it necessarily intersects the LPG litigation rule at point 4 on the midline, since the midline is the set of points that are
figure is the union of a balanced set and one that leans above the midline. This union itself
leans above the midline, establishing claim 2 of the proposition.

The argument for the case with \( \alpha < 1 \) is very similar, as can be seen via Figure 7(b). The
litigated set is the set of all cases with party beliefs lying in the triangle with vertices at points
1, 2, and 5. This triangle can be decomposed into the two shaded regions in Figure 7(b).
The lower of these regions is the 1-3-4 triangle, which lies entirely below the midline; thus
it leans below the midline. The upper shaded region is the polygon with vertices 2-3-4-5,
which can be shown to be balanced using the same proof as used for the 1-2-3-4 polygon in
Figure 7. Since the litigated set is the union of a balanced set and a set that leans below
the midline, the litigated set must lean below the midline.

\[ \square \]

D Proof of Theorem 1

Proof. Given the mass of litigated cases \( M^L \equiv \int_{[0,1]^2} L(q_d, q_p) dF_{Q_dQ_p}(q_d, q_p) \), the joint
density of litigated cases is \( f^L_{Q_dQ_p}(q_d, q_p) \equiv (M^L)^{-1} L(q_d, q_p) f_{Q_dQ_p}(q_d, q_p) \). Now define \( X^A \equiv \{(q_d, q_p) : q_p > 1 - q_d\} \) (the set of points above the midline), \( X^B \equiv \{(q_d, q_p) : q_p < 1 - q_d\} \) (the set of points below the midline), and \( X^{eq} \equiv \{(q_d, q_p) : q_p = 1 - q_d\} \) (the set of points on the midline). Let \( M^j \equiv \int_{X^j} L(q_d, q_p) dF_{Q_dQ_p}(q_d, q_p) \) for \( j \in \{A, B, eq\} \), and observe that we
can write the mass of litigated cases as \( M^L = M^A + M^B + M^{eq} \). We can write the plaintiff’s
win rate among litigated cases as

\[
W^L = \frac{1}{M^L} \left[ \int_{X^A} w(q_d, q_p) L(q_d, q_p) dF_{Q_dQ_p}(q_d, q_p) + \int_{X^B} w(q_d, q_p) L(q_d, q_p) dF_{Q_dQ_p}(q_d, q_p) + \int_{X^{eq}} w(q_d, q_p) L(q_d, q_p) dF_{Q_dQ_p}(q_d, q_p) \right].
\]

Letting \((q_{dA}, q_{pA}) \) \in \( X^A \), we must have \((q_{dB}, q_{pB}) = (1 - q_{pA}, 1 - q_{dA}) \) \in \( X^B \). Recall that
the mass of below-midline cases that are litigated is \( M^B \equiv \int_{X^B} L(q_d, q_p) dF_{Q_dQ_p}(q_d, q_p) \). Using a change of variables to \( r_d = 1 - q_d \) and \( r_p = 1 - q_p \), and observing that the inverse
image of \( X^B \) under this change of variables is \( X^A \), the change of variables formula for
integration implies that \( M^B = \int_{X^A} L(1 - r_p, 1 - r_d) dF_{Q_dQ_p}(1 - r_p, 1 - r_d) \) (note that the
Jacobian of the transformation is 1). Global simple balance of the litigation rule, implies that
\( L(q_{dA}, q_{pA}) = L(1 - q_{pB}, 1 - q_{dB}) \). By hypothesis (ii), simple balance of the joint distribution
of party beliefs with respect to \((q_{dA}, q_{pA})\), we must have \( dF(q_{dA}, q_{pA}) = dF(q_{dB}, q_{pB}) \). Thus
we have established that \( L(q_{dA}, q_{pA}) dF(q_{dA}, q_{pA}) = L(1 - q_{pB}, 1 - q_{dB}) dF(q_{dB}, q_{pB}) \). It follows
that \( M^B = \int_{X^A} L(r_p, r_d) dF_{Q_dQ_p}(r_p, r_d) = M^A \). Therefore, the overall mass of litigated cases
satisfies \( M^L = 2M^A + M^{eq} \). Again using the same change of variables as above, we may
write the contribution to the overall plaintiff’s win rate of the first two terms on the right
hand side of (7) as

their own balance pair partners, and that it also intersects the line with equation \( q_p = q_d + K/\alpha \) at point 3, 
where \( q_p = 1 \).

30
\[
\frac{1}{2M^A + M^{eq}} \int_{X^A} [w(q_d, q_p) + w(1 - q_p, 1 - q_d)]L(q_d, q_p)dF_{q_dq_p}(q_d, q_p).
\]

By complementary balance of the conditional win rate function, we have \(w(q_d, q_p) + w(1 - q_p, 1 - q_d) = 1\) for any \((q_d, q_p)\) with positive probability of litigation. Thus the double integral just above equals \(M^A\), and the first two terms on the right hand side of (7) may be written as

\[
\frac{M^A}{2M^A + M^{eq}}.
\]

If the joint distribution of party beliefs is continuous, then \(M^{eq} = 0\), since the set \(X^{eq}\) is a line in two-space, which has measure zero. In that event, the denominator of the win rate is \(2M^A\), immediately proving the theorem’s claim. Suppose instead that \(X^{eq}\) has positive measure. Since every point in \(X^{eq}\) is its own balance pair partner, complementary balance of the conditional win rate function implies that \(w(q_d, q_p) + w(1 - q_p, 1 - q_d) = 1\), so \(w(q_d, q_p) = 1/2\) for all points in \(X^{eq}\). Then the plaintiff’s win rate over \(X^{eq}\) will be \(1/2 M^{eq}\), so \(X^{eq}\) contributes \(\frac{1}{2}M^{eq}/[2M^A + M^{eq}]\) to the overall plaintiff’s win rate. Adding this to \(\frac{M^A}{2M^A + M^{eq}}\) from above, we see that the overall plaintiff’s win rate is \(\frac{M^A+(1/2)M^{eq}}{2M^A + M^{eq}}\), which is one-half, completing the theorem’s proof. \(\square\)

E  Proof of Theorem 2

Recall the definitions \(X^A \equiv \{(q_d, q_p) : q_p > 1 - q_d\}\) (the set of points above the midline), \(X^B \equiv \{(q_d, q_p) : q_p < 1 - q_d\}\) (the set of points below the midline), and \(X^{eq} \equiv \{(q_d, q_p) : q_p = 1 - q_d\}\) (the set of points on the midline); \(M^j \equiv \int_{X^j} L(q_d, q_p)dF_{q_dq_p}(q_d, q_p)\) for \(j \in \{A, B, eq\}\), with the mass of litigated cases written \(M^L = M^A + M^B + M^{eq}\).

The total mass of plaintiff wins in litigated above-midline cases is

\[
W^A \equiv \int_{X^A} w(q_d, q_p)L(q_d, q_p)dF_{q_dq_p}(q_d, q_p),
\]

while the total mass of plaintiff wins in litigated below-midline cases is

\[
W^B \equiv \int_{X^B} w(q_d, q_p)L(q_d, q_p)dF_{q_dq_p}(q_d, q_p)
= \int_{X^A} w(1 - q_p, 1 - q_d)L(1 - q_p, 1 - q_d)dF_{q_dq_p}(1 - q_p, 1 - q_d),
\]

where the second line’s equality follows because, as we have seen, \((q_d, q_p) \in X^A\) if and only if \((1 - q_p, 1 - q_d) \in X^B\). Now define \(m(q_d, q_p) \equiv L(q_d, q_p)f(q_d, q_p)\), and also define \(\lambda(q_d, q_p) \equiv m(q_d, q_p)/[m(q_d, q_p) + m(1 - q_p, 1 - q_d)]\), which always lies in the interval \([0, 1]\).

We can write the average plaintiff’s win rate among cases that do not lie on the midline as
\[
\frac{W^A + W^B}{M^A + M^B} = \int \int_{X^A} \{w(q_d, q_p)\lambda(q_d, q_p) + w(1 - q_p, 1 - q_d)[1 - \lambda(q_d, q_p)]\} \\
\times \frac{m(q_d, q_p) + m(1 - q_p, 1 - q_d)}{M^A + M^B} dQ_d dQ_p.
\]

Since \( w \) leans simply above the midline by hypothesis, we have \( w(q_d, q_p) > w(1 - q_p, 1 - q_d) \) for any \((q_d, q_p) \in X^A\). Now, the term in braces is a convex combination of \( w(q_d, q_p) \) and \( w(1 - q_p, 1 - q_d) \), so it must lie between these values. If neither \( L \) nor \( f \) leans below the midline at \((q_d, q_p)\), then \( m(q_d, q_p) \geq m(1 - q_p, 1 - q_d) \), so \( \lambda(q_d, q_p) \geq \frac{1}{2} \), with strict inequality if at least one of \( L \) or \( f \) leans simply above the midline. This implies that the term in braces will lie between \( w(q_d, q_p) \) and \( \frac{1}{2}[w(q_d, q_p) + w(1 - q_p, 1 - q_d)] \). The term in brackets is at least 1 if \( w \) does not lean complementarily below the midline; it is strictly greater than 1 if \( w \) leans complementarily above the midline. Since by hypothesis \( w \) leans both simply and complementarily above the midline, for \((q_d, q_p) \) above the midline we have \( 2w(q_d, q_p) \geq 1 + w(q_d, q_p) - w(1 - q_p, 1 - q_d) > 1 \), which implies that \( w(q_d, q_p) > \frac{1}{2} \). Therefore, if at \((q_d, q_p)\) we have

1. \( f_{Q_d q_p}(q_d, q_p) \geq f_{Q_d q_p}(1 - q_p, 1 - q_d); \)
2. \( L_{Q_d q_p}(q_d, q_p) \geq L_{Q_d q_p}(1 - q_p, 1 - q_d); \) and
3. \( w_{Q_d q_p}(q_d, q_p) \geq 1 - w_{Q_d q_p}(1 - q_p, 1 - q_d), \)

with at least one inequality satisfied strictly, the term in curly braces in (8) will exceed one-half.

Next, observe that \( \int \int_{X^A} m(q_d, q_p) dQ_d dQ_p = M^A \) and \( \int \int_{X^A} m(1 - q_p, 1 - q_d) Q_d dQ_p = M^B \). Consequently, \( \rho(q_d, q_p) \equiv [m(q_d, q_p) + m(1 - q_p, 1 - q_d)]/[M^A + M^B] \) is a probability density function. It follows that the integral in (8) can be viewed as the weighted average of values of the term in curly braces, taken over the set of above the midline points. Since each such point has a curly-brace term value above one-half, the average must also be greater than one-half.

It follows that the plaintiff’s win rate must exceed one-half among litigated cases that do not lie on the midline. If the joint density of party beliefs is continuous, then the set of cases on the midline has measure zero, and the proof is complete for the leaning-above case. Note in addition, though, that for any point that lies on the midline, the plaintiff’s win rate must be at least one-half, since, by hypothesis, there are no points where the conditional win rate function leans complementarily below the midline; thus the theorem’s hypothesis is sufficient for the plaintiff’s win rate among midline cases to be at least one-half even when there is mass on the midline.

Finally, when the inequalities in the theorem’s hypothesis are reversed, all the arguments above can be carried out in the opposite direction, with opposite effect.
F  Proof that the Plaintiff’s Win Rate Might Either Rise or Fall After a Pro-Plaintiff Change in Legal Rules

To prove the theorem it is sufficient to provide values for party beliefs in case types A, A’, G, G’, H, and H’ such that (i) the conditional win rate function has a greater value at each “prime” belief point than at the corresponding ex ante belief points, and (ii) the plaintiff’s win rate among litigated cases is lower under the ex post legal rule than under the ex ante rule.

The following table summarizes the assumed party beliefs in each ex ante/ex post pair of case types:

<table>
<thead>
<tr>
<th></th>
<th>q_d</th>
<th>q_p</th>
<th>w(q_d, q_p)</th>
<th>Litigated</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2</td>
<td>0.8</td>
<td>0.5</td>
<td>Yes</td>
</tr>
<tr>
<td>A’</td>
<td>0.3</td>
<td>0.85</td>
<td>0.575</td>
<td>Yes</td>
</tr>
<tr>
<td>G</td>
<td>0.5</td>
<td>0.85</td>
<td>0.675</td>
<td>Yes</td>
</tr>
<tr>
<td>G’</td>
<td>0.6</td>
<td>0.9</td>
<td>0.75</td>
<td>No</td>
</tr>
<tr>
<td>H</td>
<td>0.3</td>
<td>0.6</td>
<td>0.45</td>
<td>No</td>
</tr>
<tr>
<td>H’</td>
<td>0.35</td>
<td>0.75</td>
<td>0.55</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Now suppose that under both the ex ante and the ex post legal rules, the conditional win rate function at a belief point is the simple mean of the parties’ beliefs: \( w(q_d, q_p) = (q_d+q_p)/2 \). Suppose also that there are equal numbers of cases of types A, G, and H under the ex ante legal rule, and similarly equal numbers of cases of types A’, G’, and H’ under the ex post rule. Then under the ex ante rule, the plaintiff’s win rate among litigated cases is the simple mean of the conditional win rate function for cases of types A and G; this works out to 0.5875. Under the ex post rule, the plaintiff’s win rate among litigated cases is the simple mean of the conditional win rate function for cases of types A’ and H’; this works out to 0.5625. Thus in this example the plaintiff’s win rate falls after a pro-plaintiff change in the legal rule.

If we move A’ or H’ enough to the northeast, or if we move A or G slightly to the southwest (or both), we will have a greater plaintiff’s win rate under the ex post rule. For example, moving A’ to the point \((q_d, q_p) = (0.4, 0.9)\) would increase the plaintiff’s win rate among cases litigated under the ex post rule from 0.5625 to 0.6, enough for the observed plaintiff’s win rate to increase following the pro-plaintiff change in the legal rule. A smaller move, say, to \((q_d, q_p) = (0.36, 0.89)\), would yield a plaintiff’s win rate among cases litigated ex post of 0.5875—exactly equal to the win rate among cases litigated ex ante.

This proves that any direction of change in the plaintiff’s win rate is possible when there is a pro-plaintiff change in the legal rule. By regarding the “prime” cases as the ex ante distribution and the non-“prime” cases as the ex post distribution, we have the same result for a pro-defendant change in the legal rule. This proves the theorem.
G Derivation of the Reduced Form of the Simulated Priest-Klein Model

[MIGHT NEED A BIT OF CLEANING UP/TYPOS MIGHT NEED FIXING]

Define

\[ U \equiv \frac{\epsilon_p + Y - y^*}{\sigma} \quad V \equiv \frac{\epsilon_d + Y - y^*}{\sigma} \quad Z \equiv \frac{Y - y^*}{\sigma} \]

Now, \((U, V, Z)\)' may be written as the following affine transformation of \((\epsilon_p, \epsilon_d, Y)\):

\[
\begin{pmatrix}
U \\
V \\
Z
\end{pmatrix} = \frac{-y^*}{\sigma} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + T \begin{pmatrix} \epsilon_p \\ \epsilon_d \\ Y \end{pmatrix}, \quad \text{with} \quad T \equiv \begin{pmatrix} \frac{1}{\sigma} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.
\]

Since \((\epsilon_p, \epsilon_d, Y)\)' has a joint normal distribution, any affine transformation of it is joint normal, too. Thus \((U, V, Z)\)' is joint normal, with mean and variance given by\(^{26}\)

\[
E \begin{pmatrix} U \\ V \\ Z \end{pmatrix} = \frac{-y^*}{\sigma} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad Var \begin{pmatrix} U \\ V \\ Z \end{pmatrix} = \begin{pmatrix} 1 + \sigma^2 & 1 & 1 \\ 1 & 1 + \sigma^2 & 1 \\ 1 & 1 & 1 \end{pmatrix}.
\]

G.1 The conditional win rate function

Now consider the win rate conditional on \((U, V)\). From the joint normality of \((U, V, Z)\), the conditional density of \(z\) given \((U, V) = (u, v)\) can be shown to be normal with conditional mean and variance

\[ \mu_z(u, v) = \frac{1}{2 + \sigma^2} (u + v - \sigma y^*) \quad \text{and} \quad Var_z(u, v) = \frac{1}{2 + \sigma^2}. \]

By construction, the plaintiff would win in litigation whenever \(Y > y^*\). This is equivalent to the condition that \(Z > 0\), so the conditional win rate satisfies \(w(u, v) = P(Z > 0|U = u, V = v)\). Since \(|Z - \mu_z(U, V)|/\sqrt{Var_z(U, V)} \sim N(0, 1)\), we can write the conditional win rate as

\(^{26}\)This follows because

\[ Var[(U, V, Z)'] = TV(\epsilon_p, \epsilon_d, Y)T', \]

and \(Var(\epsilon_p, \epsilon_d, Y)\) is diagonal with \(Var[\epsilon_p] = Var[\epsilon_d] = \sigma^2\) and \(Var[Y] = 1\).
\[ w(u, v) = 1 - \Phi \left( -\mu_z(u, v) / \sqrt{\text{Var}_z(u, z)} \right) \\
= \Phi \left( \mu_z(u, v) / \sqrt{\text{Var}_z(u, z)} \right) \\
= \Phi \left( \frac{1}{(2 + \sigma^2)^{1/2}} [u + v - \sigma y^*] \right). \]

Under the assumptions in the simulated PK model, \( q_d = \Phi(v) \) and \( q_p = \Phi(u) \), so \( v = \Phi^{-1}(q_d) \) and \( u = \Phi^{-1}(q_p) \). Thus we have the conditional win rate function I used in the main text:

\[ w(q_d, q_p) = \Phi \left( \frac{1}{(2 + \sigma^2)^{1/2}} [x(q_d, q_p) - \sigma y^*] \right) \]

where

\[ x(q_d, q_p) \equiv \Phi^{-1}(q_d) + \Phi^{-1}(q_p). \]

Note that the rotational symmetry of the standard normal distribution implies \( \Phi^{-1}(q) = -\Phi^{-1}(1-q) \), so \( x(q_d, q_p) = -[\Phi^{-1}(1-q_d) + \Phi^{-1}(1-q_d)] = -x(1-q_p, 1-q_d). \)

It is straightforward to show that when \( y^* = 0 \), \( w(q_d, q_p) = 1 - w(1-q_p, 1-q_d) \), i.e., \( w \) globally satisfies complementary balance when \( y^* = 0 \). When \( y^* < 0 \), we have \( w(q_d, q_p) = \Phi \left( \frac{1}{(2 + \sigma^2)^{1/2}} [-x(1-q_p, 1-q_d) - \sigma y^*] \right) \) which equals \( 1 - \Phi \left( \frac{1}{(2 + \sigma^2)^{1/2}} [x(1-q_p, 1-q_d) + \sigma y^*] \right) \).

This quantity exceeds \( 1 - w(1-q_p, 1-q_d) \) whenever \( y^* < 0 \) and is less than \( 1 - w(1-q_p, 1-q_d) \) whenever \( y^* > 0 \), due to the fact that \( \Phi \) is increasing. That establishes that in the simulated Priest-Klein model, the conditional win rate function leans complementarily above when \( y^* < 0 \) and leans complementarily below when \( y^* > 0 \).

**G.2 The density of litigated cases**

After some tedious algebra, \( (U, V) \) can be shown to have the marginal density function

\[ f_{U,V}(u, v) = A(y^*, \sigma) \times \exp \left\{ - \frac{1 + \sigma^2}{2 \sigma^2 (2 + \sigma^2)} \left[ (1 + \sigma^2)(u^2 + v^2) - 2uv + 2y^* \sigma [u + v] \right] \right\}, \]

where

\[ A(y^*, \sigma) \equiv \frac{1}{2\pi \sigma \sqrt{2 + \sigma^2}} \exp \left\{ \frac{-y^2}{(2 + \sigma^2)(1 + \sigma^2)} \right\} \tag{9} \]

Now change variables from \( u \) and \( v \) to \( q_p = \Phi(u) \) and \( q_d = \Phi(v) \). Using the change of variables formula, under which the Jacobian of the transformation is \( C(q_d, q_p) \) defined below, it follows that the marginal density function of \( (q_d, q_p) \) is
\[f_{Q_d Q_p}(q_d, q_p) = \frac{D(y^*, \sigma)}{C(q_d, q_p)} \exp \left\{ -\frac{B(q_d, q_p)}{2(2 + \sigma^2)} \right\}\]

where

\[
B(q_d, q_p) \equiv (1 + \sigma^2)(\Phi^{-1}(q_p)^2 + \Phi^{-1}(q_d)^2) - 2\Phi^{-1}(q_d)\Phi^{-1}(q_p) + 2y^*\sigma[\Phi^{-1}(q_p) + \Phi^{-1}(q_d)]
\]

\[
C(q_d, q_p) \equiv \phi(\Phi^{-1}(q_d))\phi(\Phi^{-1}(q_p)),
\]

and \(D(y^*, \sigma)\) is complicated but does not depend on either party’s belief. Some tedious algebra shows that \([C(q_d, q_p)]^{-1} \exp \left\{ -\frac{B(q_d, q_p)}{2(2 + \sigma^2)} \right\}\) can be reduced, such that

\[
f_{Q_d Q_p}(q_d, q_p) = D(y^*, \sigma) \times \exp \left[ \frac{1}{2\sigma^2(2 + \sigma^2)} \left( [\Phi^{-1}(q_p) - \Phi^{-1}(q_d)]^2 - 2\sigma^2\Phi^{-1}(q_d)\Phi^{-1}(q_p) + 2(1 + \sigma^2)y^*\sigma[\Phi^{-1}(q_d) + \Phi^{-1}(q_p)] \right) \right],
\]

as provided in the main text. Since \(A(y^*, \sigma)\) does not depend on \(q_d\) or \(q_p\), it cancels out when we divide the density by any integral of it over a subset of the support of \((q_d, q_p)\) having positive measure. Consequently, provided that \(A(y^*, \sigma) > 0\)—a sufficient condition for which is \(\sigma > 0\)—the density of \((q_d, q_p)\) among litigated cases may be written

\[
f_{Q_d Q_p}^L(q_d, q_p) = \frac{\bar{f}(q_d, q_p)}{M^L},
\]

where

\[
\bar{f}(q_d, q_p) = \exp \left[ \frac{1}{2(2 + \sigma^2)} \left( [\Phi^{-1}(q_p) - \Phi^{-1}(q_d)]^2 - 2\sigma^2\Phi^{-1}(q_d)\Phi^{-1}(q_p) - 2(1 + \sigma^2)y^*\sigma[\Phi^{-1}(q_d) + \Phi^{-1}(q_p)] \right) \right],
\]

\[
M^L \equiv \int_L \bar{f}(q_d, q_p) dq_d dq_p.
\]

Thus, the level sets of \(f_{Q_d Q_p}(q_d, q_p)\) are given by the relationship \(\kappa = \lambda(q_d, q_p)\), where

\[
\lambda(q_d, q_p) \equiv [\Phi^{-1}(q_p) - \Phi^{-1}(q_d)]^2 - 2\sigma^2\Phi^{-1}(q_d)\Phi^{-1}(q_p) - 2y^*\sigma[\Phi^{-1}(q_d) + \Phi^{-1}(q_p)], \quad (10)
\]

for a family of fixed \(\kappa\) values. When \(\sigma y^* = 0\), the third term drops out of the level sets. By the rotational symmetry of the standard normal distribution, the first term satisfies
\[ h_1(q_d, q_p) \equiv [\Phi^{-1}(q_p) - \Phi^{-1}(q_d)]^2 \]
\[ = [-\Phi^{-1}(1 - q_p) + \Phi^{-1}(1 - q_d)]^2 \]
\[ = [\Phi^{-1}(1 - q_d) - \Phi^{-1}(1 - q_p)]^2 \]
\[ = h_1(1 - q_p, q - q_d), \]

so that \((q_d, q_p)\) and \((1 - q_p, 1 - q_d)\) belong to the same level set when \(\sigma y^* = 0\). This establishes that the joint density of beliefs globally satisfies simple balance when \(y^* = 0\).

Next, recall that \(x(q_d, q_p) \equiv \Phi^{-1}(q_d) + \Phi^{-1}(q_p)\), so that the third term in the definition of \(\lambda\) is \(2(1 + \sigma^2)\sigma y^* x(q_d, q_p)\). Observe that \(\lambda(q_d, q_p) = h_1(q_d, q_p) - h_2(q_d, q_p) - 2y^* \sigma x(q_d, q_p)\), where \(h_2(q_d, q_p) \equiv 2\sigma^2\Phi^{-1}(q_d)\Phi^{-1}(q_p)\). By the rotational symmetry of \(\Phi\), we have \(h_2(q_d, q_p) = 2\sigma^2\Phi^{-1}(q_d)\Phi^{-1}(q_p) = 2\sigma^2\Phi^{-1}(1 - q_p)\Phi^{-1}(1 - q_d) = h_2(1 - q_p, 1 - q_d)\); thus \(h_2\) satisfies global simple balance. Since \(h_1\) and \(h_2\) are globally simply balanced, \(\lambda(q_d, q_p) - \lambda(1 - q_p, 1 - q_d) = -2y^* \sigma [x(q_d, q_p) - x(1 - q_p, 1 - q_d)]\), which equals \(-4y^* \sigma x(q_d, q_p)\) given that \(x(q_d, q_p) = -x(1 - q_d, 1 - q_p)\). Given the definition of \(x\), it will be positive when evaluated at points that lie above the midline and negative when evaluated below the midline. Consequently, for above-midline point \((q_d, q_p)\), \(\lambda(q_d, q_p) - \lambda(1 - q_p, 1 - q_d)\) is positive when \(y^* < 0\), zero when \(y^* = 0\), and negative when \(y^* > 0\). It follows that above-midline point \((q_d, q_p)\) belongs to a higher level set of the density function than does \((1 - q_p, 1 - q_d)\) when \(y^* < 0\), the same level set when \(y^* = 0\), and a lower level set when \(y^* > 0\). This is just another way of saying that the joint density globally leans simply above the midline when \(y^* < 0\), is globally simply balanced when \(y^* = 0\), and globally leans simply below the midline when \(y^* > 0\).